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Developing a Model for Determining Practical Super Efficiency and Improvement Based on Artificial DMUs

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Abstract

In contemporary economy and society, performance analysis in the services and industries attract more and more attention. The traditional Data Envelopment Analysis (DEA) approach is an efficient method to assess the efficiency in both service and industry. But conventional DEA model has some shortages such as it doesn't suggest any improvement for efficient units and in some situations suggestions are not practical because of real conditions. In this paper we developed a model based on using artificial units to generate some improvement suggestions to efficient units which are practical. Also, we ranked units based on their supper efficiency score. Finally, we applied the model to approve the applicability of the proposed model.

Keywords: Data envelopment analysis, Artificial units, Ranking units.

1 | Introduction

Measuring the efficiency of decision making units has long been considered as a difficult task because one is dealing with complex economic and behavioral entities. This task become more difficult when it involves multiple inputs and multiple outputs, in that a set of weights has to be determined to aggregate the inputs and outputs separately to form a ration as the efficiency [1].

One approach for measuring the relative efficiencies of a set of Decision-Making Units (DMUs) which consume multiple inputs to produce multiple outputs is Data Envelopment Analysis (DEA) developed by Charnes et al [2], [3]. DEA is a mathematical programming approach to assessing relative efficiencies within a group of DMUs. Most of DEA efficiency scores vary in [0,1], the unity value being reserved to efficient

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units. In the particular case of the radial models, the CCR and the BCC models yield efficiency scores both in input and in output orientation, although non-oriented DEA efficiency scores can also be defined.

DEA suggests some improvement for inefficient units to become efficient. For instant, when the radial efficiency score of a DMU is 0.9, it is possible for this inefficient unit to reduce its inputs 10 percent to become efficient. But one of the shortages of DEA is that it doesn't recommend any improvement for efficient (strongly efficient) units. It is very desirable and requested in nowadays competitive environment to improve the efficiency of even efficient units. Improving 5-10 percent of efficient units can promote the competitive advantages of the company.

Another disadvantage of the DEA is that it recommends some improvements which aren't possible practically. Because in real environment, every input has a variation range that varies among it. For example DEA results can recommend that the first input of the DMU_o should be reduced to 100 but in practice (according to management opinion) it isn't possible to reduce it to 100. Kao [4] presented a modified model of DEA with imposed bounds to deal with this problem.

The idea of using artificial (unobserved) units first was applied by Thanassoulis and Allen [5]. They used the concept of artificial units as an alternative procedure to deal with weights restriction in DEA. The values of unobserved DMUs varied in the specific range. In another work, Sowlati and Paradi [6] used it for establishing the practical frontier. In their work, they modified a multiplier form of DEA to obtain the value of artificial units. First, they calculated the value of artificial units then by imposing some constraints such as the variation range of inputs and outputs and by adding artificial units to observation set, they derived the practical frontier. The main problem of their work is that they imposed an improvement percent to the model because the model is unbounded if it doesn't have an improvement percent. They derived the improvement percent by manager. It means that the improvement level cannot take values greater than the predetermined level.

Another challenging issue in DEA is ranking units based on their efficiency scores. There are some models have been developed to deal with this issue. Anderson and Peterson (AP) model [7] is one of the most popular methods has been applied frequently. The main problem in ranking the units in DEA is the ranking of the strongly efficient units, because we can rank inefficient and even weakly efficient units based on their radial and combined efficiency scores. In AP model, for deriving the score of a unit we delete this unit from observation set and calculate the efficiency with respect to modified Production Possibility Set (PPS). We call these models super efficiency DEA model, when a DMU under evaluation is not included in observation set. So the efficiency scores that are derived from super efficiency models may be greater than DEA scores. The problem of feasibility of super efficiency models is one of the main problems of these models. Seiford and Zhu [8] have discussed the necessary and sufficient conditions for infeasibility in super efficiency models. So, developing a feasible model to determine the super efficiency and ranking of DMUs in DEA is interesting. Chen [9] has developed some models for dealing with the problem of infeasibility in order to derive a complete ranking of DMUs and correctly capturing the super efficiency represented by input saving or output surplus. But, our presented practical super efficiency model is totally different from conventional super DEA models. First, we don’t omit any DMU from observation set to evaluate super efficiency and deriving its rank. In this model we use the concept of artificial DMUs and include these artificial DMUs to observation set. So, we derive a greater PPS and as we will prove the new PPS envelops the DEA PPS and based on it, the practical super efficiency scores are less than their efficiency scores and they won’t reach values greater than one.

In this paper, we deal with these requested issues concurrently and design a model that can generate some solutions for them. We use the concept of artificial units like Sowlati and Paradi model [6] but, there are some differences between them. In their work, first, they derive the values of inputs and outputs of artificial units by solving deferent models then by adding these artificial units with determined values to observation set, identify the practical frontier. But, in this paper, we add artificial units to observation set with unknown inputs and outputs and after solving the model we can derive the value of inputs and outputs and practical supper efficiency score simultaneously. Also, we don’t impose the improvement level to the model and the efficiency score can be discounted as much as possible.

Also we don't run the model for inefficient. Because, DEA model generates some improvement suggestions and it is not requested to derive supper efficiency score for them. And we can easily rank inefficient units. So, the proposed model has less computational complexity.

As mentioned above, proposed model has some advantages and can deal with these problems concurrently. In Section 2, we describe a conventional envelopment DEA model. In Section 3, we mention the steps should be followed to implement the model and some theorems related to the proposed model. In Section 5, we use two numerical examples to prove the applicability of the model. And finally, we end the paper with some conclusions.

2 | Basic Input Oriented Model CCR

Consider an organization consisting of several units performing similar tasks. Suppose there are J units in the organization. Each unit consumes M inputs to produce r outputs. The objective of each unit is to minimize its inputs used to produce a predetermined level of outputs. Let y_{rj} and x_{ij} be the quantity of output r produced by unit j , r=1,2,..,s , and the quantity of input i used by unit j, i=1,2,..,m, respectively. An input oriented DEA model used to calculate the efficiency score for unit o is formulated as follows:

$$
\begin{aligned}
& RE_0 = \text{Min} \quad \theta, \\
& \text{s.t.} \quad \theta x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \quad i = 1,2,..,m, \\
& \qquad y_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \quad r = 1,2,...,s, \\
& \qquad \lambda_j \geq 0,
\end{aligned}
\tag{1}
$$

where λ_j are the weights or the relative impact of unit j on the target point for unit o [8]. If the optimal solution of *Model (1)* be 1, we say that the unit o is efficient otherwise is inefficient.

3 | Practical Super DEA Model

In practical cases, it is very requested to present some advises for efficient units. But, always there are some constraints on inputs and outputs. We cannot vary the quantity of inputs and outputs as DEA results suggests. According to management opinion, every input and output has a variation range. Also there may be some relations between inputs and outputs. These variations range and relations can be determined by management.

The main aim of this paper is to present some advises for efficient units. For achieving this goal we present a model is called Super DEA model. We follow these steps to deriving the model:

Step 1. Determining DEA efficiency scores by using a conventional DEA model (an input oriented model).

In these step, we determine the efficient units. In this model we just deal with efficient models, because conventional DEA model suggests some improvement for inefficient units. We define the $\Omega = \left\{ j \middle| DMU_j \ \text{is} \ \ \text{efficient} \right\}$ as the set of efficient units.

Step 2. Deriving management opinion about the variation range of the inputs and outputs of the each efficient unit and also relations among them.

As mentioned above, in practical cases sometimes there are variation range for each input and output. For example the amount of assets and equity of a firm couldn't be less than a specific quantity. And also a firm couldn't reach revenue greater than a specific value. These quantities strongly depend on the management opinion and we should derive them by management. Therefore we have these constraints:

$$A\begin{pmatrix}\tilde{x}_{ij}\\ \tilde{y}_{rj}\end{pmatrix}\le b_j \quad j\in\Omega, \tag{2}$$

where $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are inputs and outputs of artificial DMU$_j$. All of these constraints are linear such as the upper and lower bonds of inputs and outputs and such linear relations among them. So these constraints could be shown with this matrix.

Step 3. Adding artificial DMUs to observation set.

The main idea of this model is here. In this step we add an artificial DMU to observation set corresponding to each efficient unit with unknown inputs and outputs. But, these artificial DMUs should satisfy the constraints which management determined for the inputs and outputs of the efficient units. So we have $|\Omega| = K$ artificial DMUs. Therefore we can have this *Model (2)*:

$$\begin{aligned}
SE_0 = \text{Min}\quad & \theta,\\
\text{s.t.}\quad & \theta x_{io} \ge \sum_{j=1}^{n}\lambda_j x_{ij} + \sum_{k=1}^{K}\lambda'_k \tilde{x}_{ik}, \quad i=1,2,\ldots,m,\\
& y_{ro} \le \sum_{j=1}^{n}\lambda_j y_{rj} + \sum_{k=1}^{K}\lambda'_k \tilde{y}_{rk}, \quad r=1,2,\ldots,s,\\
& A\begin{pmatrix}\tilde{x}_{ik}\\ \tilde{y}_{rk}\end{pmatrix}\le b_k, \qquad k=1,\ldots,K\\
& \sum_{j=1}^{n}\lambda_j + \sum_{k=1}^{K}\lambda'_k = 1,\\
& \lambda_j \ge 0,\ \lambda'_k \ge 0.
\end{aligned}$$

First and second constraints groups are like conventional DEA model but in this model, there are *K* artificial observations more than the conventional model. The third group of constraints can be derived from management. These constraints show that the artificial DMU$_k$ should satisfy k-the constraints. For instant, management believes that the summation of the input 1 and 2 of artificial DMU$_k$ (AD$_k$) should be greater than a specific quantity.

The *Model (2)* isn't linear because of $\lambda'_k \tilde{x}_{ik}$ and $\lambda' \tilde{y}_{rk}$. But by defining new variables $p_{ik} = \lambda'_k \tilde{x}_{ik}$ and $q_{rk} = \lambda'_k \tilde{y}_{rk}$, the *Model (2)* is converted to this linear model:

$$\begin{aligned}
SE_o = \text{Min}\quad & \theta,\\
\text{s.t.}\quad & \theta x_{io} \ge \sum_{j=1}^{n}\lambda_j x_{ij} + \sum_{k=1}^{K}p_{ik}, \qquad i=1,\ldots,m\\
& y_{io} \le \sum_{j=1}^{n}\lambda_j y_{ij} + \sum_{k=1}^{K}q_{rk}, \qquad r=1,\ldots,s\\
& \lambda'_k\, b_k \le A\begin{pmatrix}p_{ik}\\ q_{rk}\end{pmatrix}\le \lambda'_k\, b_k, \quad k=1,\ldots,K\\
& \sum_{j=1}^{n}\lambda_j + \sum_{k=1}^{K}\lambda'_k = 1,\\
& \lambda_j,\ \lambda'_k \ge 0 \qquad j=1,\ldots,n;\, k=1,\ldots,K.
\end{aligned} \tag{3}$$

The model is linear and can be solved simply by linear softwares. The variables of *Model (3)* are $(\theta,\lambda_j,\lambda'_k,p_{ik},q_{rk})$; $j=1,2,\ldots,n; i=1,2,\ldots,m; r=1,2,\ldots,s; k=1,\ldots,K$. This model has $(m+s+1)K$ variables more than conventional DEA model.

Theorem 1. *Model (3)* is always feasible.

Proof: let

$$\theta = 1, \lambda_0 = 1, \lambda_j = 0,\ \ j = 1,2,..,n \wedge j \neq o,$$
$$p_{ik} = 0, q_{rk} = 0, \lambda'_k = 0 \text{ for }\ i = 1,2,...,m \wedge r = 1,2,...,s \wedge k = 1,2,...,K.$$

It is clear that the above solution is a feasible solution of *Model (3)*.

Theorem 2. If W be a feasible solution of *Model (1)*, then (W, 0) is a feasible solution of *Model (3)*.

Proof: let $W = \left(\tilde{\theta}, \lambda_j ; j = 1,2,...,n\right)$ be a feasible solution of *Model (1)*. It is clear that:

$$W' = \left(\tilde{\theta}, \lambda_j; j = 1,2,...,n, \lambda'_k = 0, p_{ik} = 0, q_{rk} = 0; i = 1,2,...,m \wedge r = 1,2,...,s \wedge k = 1,2,...,K\right).$$

Is a feasible solution of *Model (3)*.

So we have $\text{for all } W \in S \Rightarrow (W,0) \in S'$.

Lemma 1. The optimum solution of *Model (3)* (SE_o) isn't greater than the optimum solution of *Model (1)* (RE_o):

$$SE_o \leq RE_o.$$

Proof: let W be a feasible solution of *Model (1)* and RE its corresponding objective function. Based on *Theorem 2*, we have (W,0) is a feasible solution of *Model (3)* and its corresponding objective function is SE. it is clear that SE and *RE* have the same value (SE=RE). Because RE_o and SE_o are optimum solution of *Models (1)* and *(3)* respectively, we can conclude that $SE_o \leq SE = RE \text{ for all } W \in S$, so the optimum value of *Model (1)* is greater than SE_o:

$$SE_o \leq RE_o.$$

Now, we summarize the proposed model. In *Fig. 1*, we mention a stepwise procedure should be followed to apply the model

4 | Application

4.1 | Numerical example 1

In this section for approving applicability of the proposed model we assess the efficiency and the practical super efficiency of hospitals. These hospitals use 2 inputs, number of doctors (I_1) and number of nurses (I_2) to provide 2 outputs, number of outpatients (O_1) and number of inpatients (O_2). Data have been shown in *Table 1*.

First, we determined the efficiency scores of units and determined also strong efficient units. According to the model, units A and E are strongly efficient and there is no improvement suggestion for these units. So, , and we should calculate the super efficiency scores for these units to achieve some improvement for these units. Next step is gathering management opinion about the inputs and outputs variation of units A and D. finally by adding these constraints and artificial units to the model and solving the model separately for each unit A and D, we calculate the super efficiency of these units.

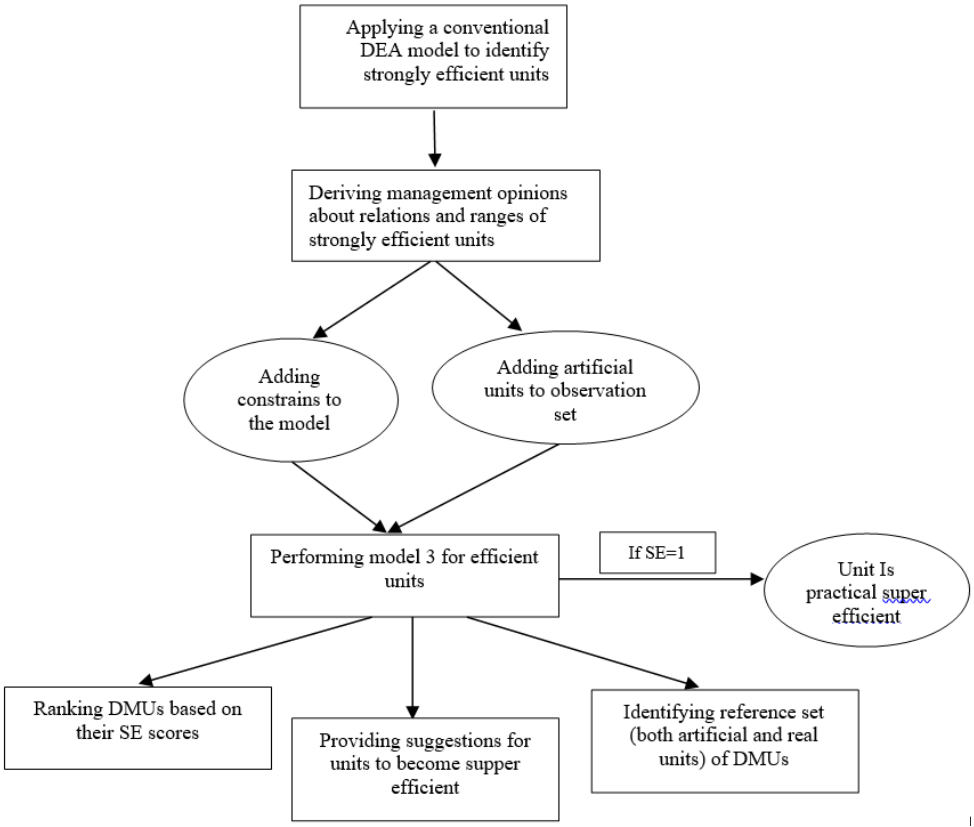

Fig. 1. Steps should be followed to apply the model.

Table 1. Data and results of example 1.

DMU	I_1	I_2	O_1	O_2	EFFICIENCY	Practical Super Efficiency	Rank
A	20	151	100	90	Strongly efficient	0.9578	2
B	19	131	150	50	Weakly efficient	-	
C	25	160	160	55	Weakly efficient	-	
D	27	168	180	72	strongly efficient	0.9764	1
E	22	158	94	66	Inefficient	-	
F	55	255	230	90	Inefficient	-	
G	33	235	220	88	Inefficient	-	

The *Model (3)* for deriving supper efficiency of unit A is as below:

$$
\begin{aligned}
&\text{Min } \theta,\\
&\text{s.t. } 20\theta - 20\lambda_1 - 19\lambda_2 - 25\lambda_3 - 27\lambda_4 - 22\lambda_5 - 55\lambda_6 - 33\lambda_7 - p_{11}\lambda - p_{12} \geq 0,\\
&151\theta - 151\lambda_1 - 131\lambda_2 - 160\lambda_3 - 168\lambda_4 - 158\lambda_5 - 255\lambda_6 - 235\lambda_7 - p_{21} - p_{22} \geq 0,\\
&100\lambda_1 + 150\lambda_2 + 160\lambda_3 + 180\lambda_4 + 94\lambda_5 + 230\lambda_6 + 220\lambda_7 + q_{11} + q_{12} \geq 100,\\
&90\lambda_1 + 50\lambda_2 + 55\lambda_3 + 72\lambda_4 + 66\lambda_5 + 90\lambda_6 + 88\lambda_7 + q_{21} + q_{22} \geq 90,\\
&-320\lambda'_1 + p_{11} + 2p_{21} \geq 0 \;,\; 195\lambda'_1 - q_{11} - q_{21} \geq 0,\\
&-18\lambda'_1 + p_{11} \geq 0,\; -145\lambda'_1 + p_{21} \geq 0,\; 22\lambda'_1 - p_{11} \geq 0,\; 155\lambda'_1 - p_{21} \geq 0,\\
&-95\lambda'_1 + q_{11} \geq 0,\; 105\lambda'_1 - q_{11} \geq 0,\; -85\lambda'_1 + q_{21} \geq 0,\; 95\lambda'_1 - q_{21} \geq 0,\\
&322\lambda'_1 - p_{11} - 2p_{21} \geq 0,\; -180\lambda'_1 + q_{11} + q_{21} \geq 0,\\
&-20\lambda'_2 + p_{12} \geq 0,\; 22\lambda'_2 - p_{12} \geq 0,\; -153\lambda'_2 + p_{22} \geq 0,\; 160\lambda'_2 - p_{22} \geq 0,\\
&-90\lambda'_2 + q_{12} \geq 0,\; 100\lambda'_2 - q_{12} \geq 0,\; -60\lambda'_2 + q_{22} \geq 0,\; 70\lambda'_2 - q_{22} \geq 0,\\
&-340\lambda'_2 + p_{12} + 2p_{22} \geq 0,\; 165\lambda'_2 - q_{12} - q_{22} \geq 0,\\
&\theta \text{ free, the rest Variables are nonnegative.}
\end{aligned}
$$

After solving this linear model we obtained the optimum result of the model. Then by transmitting the results, we can convert the results to obtain value of artificial units. Results have been shown in *Table 2*.

Table 1. Value of artificial units (Reference set of DMU A and D).

DMU	SE	λ_1'	λ_2'	$\tilde{x}_{11}$	$\tilde{x}_{21}$	$\tilde{y}_{11}$	$\tilde{y}_{21}$	$\tilde{x}_{12}$	$\tilde{x}_{22}$	$\tilde{y}_{12}$	$\tilde{y}_{22}$
A	0.9578	0.9091	0.050	19.5	150	100	95	30	167	180	80
D	0.9764	0	0.969	-	-	-	-	27	169	186	75

The practical supper efficiency of unit A is about 0.957 and it is less than 1. So, it is possible to this unit to decrease its inputs to be supper efficient. The reference set of unit A includes 2 artificial units $(\tilde{x}_1, \tilde{y}_1)$ and $(\tilde{x}_2, \tilde{y}_2)$. These artificial units with known inputs and outputs are references for unit A. and unit A can benchmark them to achieve more efficiency score. In the same way, we derived the results for unit D. results have been shown in *Table 2*.

Results show that the only reference unit for unit D is an artificial unit with known value (27,169, 186, 75). The practical supper efficiency of unit D is 0.9746 and it is possible for this unit to decrease its inputs (1-0.9746=0.0254) percent to become practical supper efficient and for reaching this aim it can benchmark its reference set with practical inputs and outputs value. Now we can rank units A and D based on their SE scores. Results have been shown in *Table 1*.

4.2 | Numerical Example 2

In this section we use the data of Chen's work [9] that is about evaluating and ranking of Japanese companies. The procedure is like previous example. At first, we obtain the efficient DMUs which are, $\Omega = \{DMU_1, DMU_2, DMU_6, DMU_8, DMU_{18}\}$. Then we imposed some constraints were about the variation range of inputs and outputs such as lower band of each input for each company and relations among inputs. For example, the summation of assets and equity should be greater than a specific quantity. By following the procedure shown in *Fig. 1* we derived these results:

Table 3. Data and results.

DMU	Asset	Equity	Employee	Revenue	$\theta_{VRS}{}^*$	$\theta_{VRS}{}^{AP}$	Rank	SE	SE Rank
1	50,905.30	5,137.90	40,000.00	106,793.20	1	2.366	3	0.9885	1
2	51,432.50	2,333.80	5,775.00	106,184.10	1	6.692	1	0.9858	2
3	67,553.20	7,253.20	36,000.00	104,656.30	0.74248				
4	112,698.10	47,177.00	183,879.00	97,387.60	0.4108				
5	49,742.90	2,704.30	5,844.00	91,361.00	0.91739				
6	41,168.40	4,351.50	30,700.00	86,921.00	1	1.2091	4	0.9815	3
7	133,008.80	47,467.10	138,150.00	74,323.40	0.26865				
8	35,581.90	1,274.40	19,461.00	66,144.00	1	1.1458	5	0.9609	5
9	73,917.00	21,914.20	328,351.00	60,937.90	0.40528				
10	60,639.00	26,988.40	282,153.00	58,361.60	0.47569				
11	48,117.40	13,930.70	177,000.00	51,903.00	0.54156				
12	52,842.10	9,583.60	39,467.00	50,263.50	0.47975				
13	38,455.80	13,473.80	112,200.00	47,597.70	0.62931				
14	46,013.00	8,023.30	198,000.00	40,492.70	0.45933				
15	39,052.20	8,901.60	1,888,000.00	4,050.30	0.53631				
16	110,055.80	12,157.70	50,558.00	38,869.50	0.18567				
17	38,015.00	6,517.40	157,773.00	36,356.40	0.50901				
18	16,696.00	676.10	3,654.00	30,205.30	1	2.899	2	0.9786	4
19	17,023.60	10,816.60	31,000.00	29,612.20	0.980706				
20	31,997.00	4,129.60	116,479.00	28,982.20	0.5218				

As we can see, practical rank of DMUs (ranking based on SE scores) is totally different from ranking based on Chen's super efficiency model. Because, in practice we considered the conditions that every artificial unit should satisfy them. After ranking DMUs, we obtain the reference set of each efficient DMU with specific value of inputs and outputs. Results have been shown in *Table 4*. These reference sets and DMUs are all practical because, all of their inputs and outputs are in the range of variation.

6 | Conclusion

Nowadays companies operate in an extremely competitive environment and the improvement of the efficiency is on of the challenging issues that companies should go on it. As a shortage of the DEA, it assesses the efficiency in the best conditions and it is possible that an efficient unit become inefficient when we assess the efficiency by another methods. So, the efficiency improvement of efficient units is a requested subject in DEA. Also sometimes, the improvement suggestions which derived by DEA are not practical. In this paper we developed a model to deal with these problems. Proposed model assessed the super efficiency of efficient units to generate some improvement suggestions to them. The model is based on the concept of artificial units to enlarge the observation set and PPS. Results showed that artificial units with known inputs and outputs play as the reference units of the efficient units. These artificial units are practical because we imposed the variation range of their inputs and outputs also the relations among them. So, they can be benchmarked practically. And as the marginal results of the proposed model we could rank them based on their supper efficiency scores. Proposed model has some advantages that it is feasible, in spit of AP model, and is thoroughly practical and the improvement suggestions can be applied by units completely. As the model is strongly based on subjective judgments of management, for future works, we can use fuzzy logic to handle the ambiguity of subjective judgments more efficiently.

Table 4. Practical Reference set of efficient DMUs.

		DMU_1	DMU_2	DMU_6	DMU_8	DMU_{18}
$\tilde{\lambda}_1$		0.9997	0.0002	0	0	0
$\tilde{\lambda}_2$		0.0003	0.9998	0.099	0.1707	0
$\tilde{\lambda}_3$		0	0	0.8673	0	0.0000034
$\tilde{\lambda}_4$		0	0	0	0.6392	0
$\tilde{\lambda}_5$		0	0	0.0336	0.1901	0.999999
Reference set	$ADMU_1$	(50318, 5079.5, 59550, 106789)	(41678.5,4501.5,31675,890195)	-	-	-
	$ADMU_2$	(56127, 2778.6, 6113,104131.4)	(50701.6,2300.1,5500.1,106187.5)	(50520, 2500.9,49139, 106226)	(50786.4,2199.4,5498, 106155)	-
	$ADMU_3$	-	-	(40201, 4600.1,29000, 86922.3)		(40231,4173.9,31000,86144.9)
	$ADMU_4$	-	-		(35051,1150,18000,66145.9)	
	$ADMU_5$	-	-	(16017.9,1001.1,3403.8,30238.9)	(35051,1150,18000,66145.9)	(16339,665,3410,30210)

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Data Availability

The data used in this study are available upon reasonable request from the corresponding author. Any supporting materials that are not publicly accessible can be shared with interested researchers for academic and research purposes.

Conflicts of Interest

The authors declare that there are no conflicts of interest related to this study.

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