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Fuzzy Mathematical Programming Approach for the Design of Integrated Production and Distribution Systems in Supply Chain Network

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Abstract

Supply Chain Management (SCM) is a mathematical approach to control the supply chain in an efficient way. Traditional SCM models require crisp data; however, in real-world problems the data available may often be imprecise like fuzzy numbers. In order to enter the fuzzy numbers into SCM models many attempts has been made by the researchers but in the existing approaches many information on uncertainties are lost. This paper represents a method which keeps the uncertainty of the data through the computation. In the presented method, keeping the uncertainty through the computation yields a multi objective nonlinear programming.

Keywords: Supply chain management, Fuzzy numbers, Multi objective nonlinear programming.

1 | Introduction

Supply Chain Management (SCM) is the process of planning, implementing and controlling the operations of the Supply Chain (SC) in an efficient way. SCM spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption [1]. Production and distribution operations are two key business functions in SCM. To achieve optimal operational performance in a SC, it is critical to integrate these two functions and schedule them jointly in a coordinated manner. For the most part, these two functions have been studied independently of each other leading to globally sub-optimal decisions [2].

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A critical review carried out, measures the relationships between SC integration and performance: a high level of integration has a positive impact on corporate and SC performance. So, in this paper an integrated production-distribution model consisting multi periods and multi products and considering a plant, distribution centers (DCs) and customer zones is presented.

Whereas, SCs are generally complex and are described by numerous activities spread over multiple functions and organizations, which raise interesting challenges for effective SC coordination and as regards SC, is a dynamic network of several business entities that involve a high degree of imprecision. Most studies have focused on traditional analytical and heuristic methods to model the SC network planning problem.

A few studies have attempted to model the problem in an uncertainty environment. Likewise, stochastic models in such conditions may not lead to fully satisfactory results, so using fuzzy models allows us to remove this drawback [3]. However, the majority of existing fuzzy models consider only separate aggregate production planning without taking into account the interdependent nature of production and distribution systems. This limited approach often leads to inadequate results. This is mainly due to its real-world character, where uncertainties in activities extending from the suppliers to the customers make SC imprecise. Therefore fuzzy set theory is a suitable tool to come up with such a complicated system [4].

In this paper, we present a model formulation for application in the consumer goods industry which integrates the production and distribution plans taking into account the fuzziness.

This model is used to optimize the quantity of each product produced in plant and inventory level of products in each DC, transportations between plants, DCs and customer zones when costs are available in fuzzy format.

The rest of this paper is organized as follows. Section 2 presents a literature review about fuzzy applications in production-distribution planning. Section 3 is devoted to problem formulation in deterministic condition and we review briefly fuzzy set theory. The fuzzy mixed-integer programming (FMIP) model is proposed in Section 4. In Section 5, the behavior of the model is evaluated. This is followed by some concluding remarks is Section 6.

2 | Literature Review

The production and distribution systems have been considered the main processes in SC. These systems traditionally were viewed as different functions in the firms and usually optimized seperately or sequentially. In two recent decades, an integrated view of production and distribution systems have received considerable attention regarding its ability to help enterprises to make effective tactical and operational decisions and to compete effectively in the marketplace by optimizing different functions simultaneously. Many researchers, [5-10], provide goog review of the literature related to the integrated analysis of the production and distribution systems in supply chan. In this way, Lee and Kim [11] develop a hybrid simulation-analytic approach to solve an integrated production-distribution planning problem in SC. Review an industrial example and present a mixed integer program model to deal with the integrated production-distribution problem. [12] investigate the differences between the values of coordinting production and distribution planning and those of production scheduling and distribution scheduling solved separately. [13] used a new solution procedure for an integrated production-distribution planning problem consistiong Lagrangian Relaxation and a heuristic algorithm. Lee and Kim [14] and Lee and Kim [11] propose a hybrid approach that combines analytic and simulation method for production–distribution planning in SC.

By seeking the literature of SC, limited number of research that use fuzzy set theory is found. Petrovic et al. [15] developed a fuzzy generative SC model to determine target order-up-to levels of inventories under uncertain demand and external supply of raw materials. Giannoccaro et al. [16] presented a methodology to define a three-stage SC inventory management policy which the echelon stock concept was adopted to manage the SC inventory in an integrated manner, whereas fuzzy set theory was used to properly model the uncertainty associated with both market and inventory costs. Xie et al. [17] presented a new hierarchical, two-level approach to inventory management and control in SCs. They supposed that the SC operates under uncertainty in customer demand, which is described by imprecise terms and modelled by fuzzy sets. Chen et al. [18] present a fuzzy decision-making approach to deal with the supplier selection problem in SC system. Chang et al. [19] propose a fuzzy multiple attribute decision making method based on the fuzzy linguistic quantifier to select SC artners at different phases of product life cycle. [4] presented Five crisp and fuzzy models for supp SC of an automotive manufacturing system. Aliev et al. [3] developed a fuzzy integrated multi-period and multi-product production and distribution model in SC. The model is formulated in terms of fuzzy programming and the solution is provided by genetic optimization. Xu et al. [20] proposed a random fuzzy programming model and a methodology for solving a multi-stage SC design problem of a realistic scale in the random fuzzy environment. Based on the concept of random fuzzy variable. They used probability theory, fuzzy set theory and optimizing theory as their research tool. They presented novel technique called spanning-tree based on genetic algorithm to get the heuristic solutions. Selim et al. [21] asserted that fuzzy goal programming approaches can effectively be used in handling the collaborative production–distribution planning problems in both centralized and decentralized SC structures. Liang and Ceng [22] presented a systematic framework that facilitates fuzzy decision-making for solving the multi-objective manufacturing/distribution planning decision problems with multi-product and multi-time period in SC s under an uncertain environment, enabling the decision maker to adjust the search direction during the solution procedure to obtain a preferred satisfactory solution. Bilgen [23] provided a model formulation for application in the consumer goods industry consisting of multiple manufacturers, multiple production lines and multiple (DCs) which integrates the production and distribution plans taking into account the fuzziness in the available capacity restrictions and the aspiration level of costs. Mula et al. [24] applied a fuzzy approach to a SC production planning problem with lack of knowledge in demand data. They studied an application of known possibilistic programming in a SC planning case study. Liang [22] proposed a fuzzy mathematical programming methodology for solving manufacturing/distribution planning decision integration problems attempting to minimize the total manufacturing and distribution costs by considering the levels of inventory, subcontracting and backordering, the available machine capacity and labor levels at each source, forecast demand and available warehouse space at each destination.

3 | Priliminaries

In this section we review some basic concepts in SCM models and fuzzy sets.

3.1 | Problem Formulation

In this section, a Mixed Integer Linear Programming (MILP) model is proposed considering a 3-layer SC system: a plant, multiple DCs and multiple customer zones in multiple periods with multiple products. The mathematical model of the integrated system is developed based on network structure as shown in *Fig. 1*. In each planning horizon, the plant performs operations to produce finished products. Then the plant ships the finished goods to DCs. Finally, the finished products are distributed from DCs to customer zones.

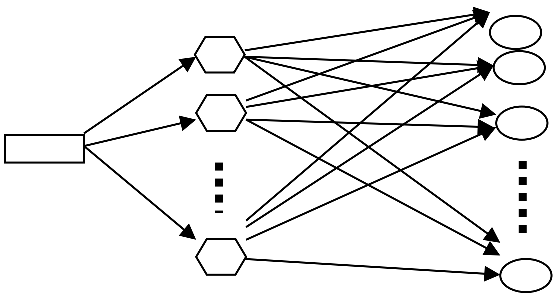

Fig. 1. The network structure of the integrated production and distribution system.

Likewise, specified capacities for plant and each DC are determined in the model, and demand of each customer zone can be satisfied from multiple DCs in the same time. Besides, the model determines assignment of each customer zone to DCs, quantity of each product produced in each period, inventory level of products which is held in each DC, and quantity of products delivered to DCs and Customer zones.

The objective function of the model is to minimize manufacturing cost, transportation costs between plant to DCs and DCs to customer zones, and inventory holding costs while satisfying all customer demands, plant capacity and DCs capacities. The mathematical model describing the characteristic of the problem can be formulated based on following variables and parameters:

3.1.1 | Notations

Sets and indices

T: Set of time periods.

t: Index for time periods.

P: Set of products.

p: Index for products

W: Set of (DCs).

w: Index for (DCs).

I: Set of customer zones.

i: Index for customer zones.

Parameters

B: Time unit available for production in any given period.

U_p: Processing time for producing one unit of product p.

SC_{pt}: Set up cost for producing product p during period t.

PC_{pt}: Variable cost to produce a unit of product p.

HC_{pw}: Inventory holding cost per unit of product p at DCw.

TC_{pw}: Cost of travel from plant to DCw per unit of product p.

TC_{pwi}: Cost of travel from DCw to customer zone i per unit of product p.

λ_{pt}: Backorder cost of product p in period t.

H_w: Holding capacity at DCw.

V_p: Volume of product p.

D_{pit}: Demand of product p at customer zone i in period t.

M: A sufficient large positive number.

Decision variables

x_{pt}: 1 if the plant should be set up for product p in period t; 0 otherwise.

q_{pt}: Quantity of product p produced in period t.

l_{pwt}: Inventory level of product p at DCw in period t.

f_{wt} : Quantity of product *p* delivered from plant to DC_w in period *t*.

g_{wit} : quantity of product p delivered from DCw to customer zone i in period t.

Mathematical model

$$\text{Min Z: } \sum_t \sum_p SC_{pt} x_{pt} + \sum_t \sum_p PC_{pt} q_{pt} \tag{1}$$

$$+\sum_t \sum_p \sum_w HC_{pw} l_{pwt} \tag{2}$$

$$+\sum_t \sum_p \sum_w TC_{pw} f_{pwt} \tag{3}$$

$$+\sum_t \sum_p \sum_w \sum_i TC_{pwi} g_{pwit}. \tag{4}$$

Subject to

$$\sum_p U_p q_{pt} \le B, \quad \text{for all t,} \tag{5}$$

$$q_{pt} \le M x_{pt}, \quad \text{for all t, p,} \tag{6}$$

$$\sum_p f_{pwt} V_p + \sum_p l_{pw(t-1)} V_p \le H_w, \quad \text{for all t, w,} \tag{7}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_i g_{pwit}, \quad \text{for all t, p, w,} \tag{8}$$

$$\sum_w f_{pwt} = q_{pt}, \quad \text{for all t, p,} \tag{9}$$

$$\sum_w g_{pwit} = D_{pit}, \quad \text{for all t, p, i,} \tag{10}$$

$$x_{pt} = 0,1, \quad \text{for all t, p,} \tag{11}$$

$$l_{pwt} \ge 0, \quad \text{for all t, p, w,} \tag{12}$$

$$f_{pwt} \ge 0, \quad \text{for all t, p, w,} \tag{13}$$

$$g_{pwit} \ge 0, \quad \text{for all t, p, w, i.} \tag{14}$$

The Objective function minimizes total costs of the system. Alternatively, *Phrase (1)* includes set up costs and variable costs of product p producing in period t. *Phrase (2)* is the holding inventory costs in DCs. *Phrases (3)* and *(4)* state transportation costs from plant to DCs and DCs to customer zones respectively.

Constraint (5) ensure that the plant capacity is respected. *Constraint (6)* guarantee that the set up cost will incur only whenever there is production in a given period t. *Constraint (7)* express the volume capacity constraints of DCs. *Constraint* (8) assure the balance of inventory level in DCs.

Constraint (9) ensure that the total quantity of product p delivered from the plant to DC_w is equal to production quantity in a given period t. *Constraint (10)* state each customer zone demands are completely satisfied. Finally *Constraints (11)-(14)* state the types of decision variables.

3.2 | Fuzzy Set and Notations

Definition 1. If *X* is a collection of objects denoted generically by *x*, then a *fuzzy set* $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}.$$

$\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in $\tilde{A}$ that maps X to the membership space M (When *M* contains only the two points 0 and 1,

$\tilde{A}$ is nonfuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

Definition 2. A fuzzy number $\tilde{A}$ is called triangular fuzzy number if its membership degree be as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - x^l}{x^m - x^l}, & x \le x^m, \\ \dfrac{x^u - x}{x^u - x^m}, & x \ge x^m. \end{cases}$$

x^m called the mean value of $\tilde{M}$, is a real number, and x^l and x^u are, respectively the lower and upper bounds of $S(\tilde{A})$. Symbolically, $\tilde{A}$ is denoted by (x^m, x^l, x^u).

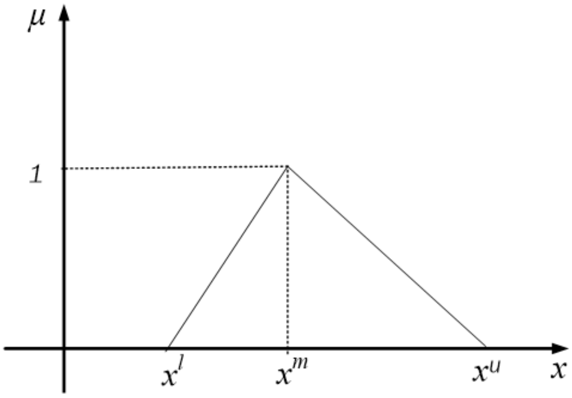

Fig. 2. Representation of a triangular fuzzy number and the boundaries of α-level set.

Definition 3. Consider the fuzzy numbers $\tilde{x}_k = (x_k{}^m, x_k{}^l, x_k{}^u)$, $(k = 1,...,n)$. we define $\tilde{x}'_j = (x'^m_j, x'^l_j, x'^u_j)$ as follows and call it normalized $\tilde{x}_j$.

$$\mu(\tilde{x}_j) = \mu(\tilde{x}'_j), \quad \tilde{x}'_j = \frac{x_j}{\max\{\tilde{x}^u_k\}}, \quad x_j{}^l \le x_j \le x^u_j.$$

4 | Fuzzy SCM Model

Let $\tilde{TC}_{pw}, \tilde{TC}_{pwi}, \tilde{HC}_{pw} \tilde{PC}_{pw}$ and $\tilde{SC}_{pt}$ are triangular fuzzy numbers the model changes as follows:

$$Min\ Z: \sum_t \sum_p \tilde{SC}_{pt} x_{pt} + \sum_t \sum_p \tilde{PC}_{pw} q_{pt} \quad (1)$$

$$+\sum_t \sum_p \sum_w \tilde{HC}_{pw} l_{pwt} \quad (2)$$

$$+\sum_t \sum_p \sum_w \sum_i \tilde{TC}_{pwi} g_{pwit} \quad (3)$$

$$+\sum_t \sum_p \sum_w \sum_i \tilde{TC}_{pwi} g_{pwit}. \quad (4)$$

Subject to

$$\sum_p U_p q_{pt} \le B \quad \text{for all } t, \quad (5)$$

$$q_{pt} \le M x_{pt}, \quad \text{for all } t, p, \quad (6)$$

$$\sum_p f_{pwt} V_p + \sum_p l_{pw(t-1)} V_p \le H_w, \quad \text{for all } t, w, \tag{7}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_i g_{pwit}, \quad \text{for all } t, p, w, \tag{8}$$

$$\sum_p U_p q_{pt} \le B, \quad \text{for all } t, \tag{9}$$

$$q_{pt} \le M x_{pt}, \quad \text{for all } t, p, \tag{10}$$

$$\sum_p f_{pwt} V_p + \sum_p l_{pw(t-1)} V_p \le H_w, \quad \text{for all } t, w, \tag{11}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_i g_{pwit}, \quad \text{for all } t, p, w, \tag{12}$$

$$\sum_w f_{pwt} = q_{pt}, \quad \text{for all } t, p, \tag{13}$$

$$\sum_w g_{pwit} = D_{pt}, \quad \text{for all } t, p, i, \tag{14}$$

$$x_{pt} = 0,1, \quad \text{for all } t, p, \tag{15}$$

$$l_{pwt} \ge 0, \text{ for all } t, p, w, \tag{16}$$

$$f_{pwt} \ge 0, \text{ for all } t, p, w, \tag{17}$$

$$g_{pwit} \ge 0, \text{ for all } t, p, w, i. \tag{18}$$

The basic idea in this approach is to transform the fuzzy SCM model into a crisp linear programming problem by applying an alternative α-cut approach. This approach assumes that the solution lies in the interval and defines suitable variables for this solution. This method used α-cut approach so it does not retain the uncertainty in formation completely. In other words, each fuzzy number is converted to an interval with the same membership in the entire interval. We propose an alternative fuzziness. The advantage of the new approach is that the uncertainty concept will be kept throughout the computation.

An alternative fuzzy SCM model under uncertainty

Model presented below keeps the imprecision of the data throughout the calculations. This model minimizes the objective function and maximizes the membership functions of fuzzy parameters.

$$\text{Max}\{\mu(SC'_{pt}), \mu(PC'_{pt}), \mu(HC'_{pw}), \mu(TC'_{pw}), \mu(SC'_{pt}), \mu(SC'_{pwi})\tag{1}$$

$$\text{Min}\{\sum_t \sum_p SC'_{pt} x_{pt} + \sum_t \sum_p PC'_{pt} q_{pt} + \sum_t \sum_p \sum_w HC'_{pw} l_{pwt} + \sum_t \sum_p \sum_w TC'_{pw} f_{pwt} + \sum_t \sum_p \sum_w \sum_i TC'_{pwi} g_{pwit}\}. \tag{2}$$

Subject to:

$$\sum_p U_p q_{pt} \le B, \quad \text{for all } t, \tag{3}$$

$$q_{pt} \le M x_{pt}, \quad \text{for all } t, p, \tag{4}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_i g_{pwit}, \quad \text{for all } t, w, \tag{5}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_i g_{pwit}, \quad \text{for all } t, p, w, \tag{6}$$

$$\sum_w f_{pwt} = q_{pt}, \quad \text{for all } t, p, \tag{7}$$

$$\sum_w g_{pwit} = D_{pit}, \quad \text{for all } t, p, i, \tag{8}$$

$$x_{pt} = 0,1, \quad \text{for all } t, p, \tag{9}$$

$$l_{pwt} \ge 0, \quad \text{for all } t, p, w, \tag{10}$$

$$f_{pwt} \ge 0, \quad \text{for all } t, p, w, \tag{11}$$

$$g_{pwit} \geq 0, \quad t, p, w, i, \tag{12}$$

$$\frac{TC^{l}{}_{pw}}{\max\left\{TC^{m}{}_{pw}\right\}} \leq TC'_{pw} \leq \frac{TC^{u}{}_{pw}}{\max\left\{TC^{m}{}_{pw}\right\}}, \tag{13}$$

$$\frac{TC^{l}{}_{pwi}}{\max\left\{TC^{m}{}_{pwi}\right\}} \leq TC'_{pwi} \leq \frac{TC^{u}{}_{pwi}}{\max\left\{TC^{m}{}_{pwi}\right\}}, \tag{14}$$

$$\frac{HC^{l}{}_{pw}}{\max\left\{HC^{m}{}_{pw}\right\}} \leq HC'_{pw} \leq \frac{HC^{u}{}_{pw}}{\max\left\{HC^{m}{}_{pw}\right\}}, \tag{15}$$

$$\frac{PC^{l}{}_{pw}}{\max\left\{PC^{m}{}_{pw}\right\}} \leq PC'_{pw} \leq \frac{PC^{u}{}_{pw}}{\max\left\{PC^{m}{}_{pw}\right\}}, \tag{16}$$

$$\frac{SC^{l}{}_{pw}}{\max\left\{SC^{m}{}_{pw}\right\}} \leq PC'_{pw} \leq \frac{SC^{u}{}_{pw}}{\max\left\{SC^{m}{}_{pw}\right\}}. \tag{17}$$

The fuzzy parameters are normalized and then considered as bounded variables in *Constraints (13)-(17)*. The membership degrees of these variables are maximized by the model in the first objective function.

In order to convert the above multi objective nonlinear programming to a nonlinear programming model, we present the following model:

$$\begin{aligned} \text{Min}(&\sum_{t}\sum_{p} SC'_{pt} x_{pt} + \sum_{t}\sum_{p} PC'_{pt} q_{pt} + \sum_{t}\sum_{p}\sum_{w} HC'_{pw} l_{pwt} + \sum_{t}\sum_{p}\sum_{w} TC'_{pw} f_{pwt} \\ &+ \sum_{t}\sum_{p}\sum_{w}\sum_{i} TC'_{pwi} g_{pwit} - \alpha). \end{aligned} \tag{1}$$

$$\sum_{p} U_p q_{pt} \leq B, \quad \text{for all } t, \tag{2}$$

$$q_{pt} \leq M x_{pt}, \quad \text{for all } p, t, \tag{3}$$

$$\sum_{p} f_{pwt} V_p + \sum_{p} l_{pw(t-1)} V_p \leq H_w, \quad \text{for all } w, t, \tag{4}$$

$$l_{pwt} = l_{pw(t-1)} + f_{pwt} - \sum_{i} g_{pwit}, \quad \text{for all } p, w, i, t, \tag{5}$$

$$\sum_{w} f_{pwt} = q_{pt}, \quad \text{for all } p, t, \tag{6}$$

$$\sum_{w} g_{pwit} = D_{pit}, \quad \text{for all } p, i, t, \tag{7}$$

$$x_{pt} = 0,1, \quad \text{for all } p, t, \tag{8}$$

$$l_{pwt} \geq 0, \quad \text{for all } p, w, t, \tag{9}$$

$$f_{pwt} \geq 0, \quad \text{for all } p, w, t, \tag{10}$$

$$g_{pwit} \geq 0, \quad \text{for all } p, w, t, \tag{11}$$

$$\frac{TC^{l}_{pwi}}{\max\{TC^{m}_{pwi}\}} \leq TC'_{pwi} \leq \frac{TC^{u}_{pwi}}{\max\{TC^{m}_{pwi}\}}, \tag{12}$$

$$\frac{TC^{l}_{pw}}{\max\{TC^{m}_{pw}\}} \leq TC'_{pw} \leq \frac{TC^{u}_{pw}}{\max\{TC^{m}_{pw}\}}, \tag{13}$$

$$\frac{HC^{l}_{pw}}{\max\{HC^{m}_{pw}\}} \leq HC'_{pw} \leq \frac{HC^{u}_{pw}}{\max\{HC^{m}_{pw}\}}, \tag{14}$$

$$\frac{PC^l_{pw}}{\max\{PC^m_{pw}\}} \leq PC'_{pw} \leq \frac{PC^u_{pw}}{\max\{PC^m_{pw}\}}, \quad (15)$$

$$\frac{SC^l_{pw}}{\max\{SC^m_{pw}\}} \leq SC'_{pw} \leq \frac{SC^u_{pw}}{\max\{SC^m_{pw}\}}, \quad (16)$$

$$\mu(SC'_{pt}) \leq \alpha, \quad (17)$$

$$\mu(PC'_{pt}) \leq \alpha, \quad (18)$$

$$\mu(HC'_{pw}) \leq \alpha, \quad (19)$$

$$\mu(TC'_{pw}) \leq \alpha, \quad (20)$$

$$\mu(SC'_{pwi}) \leq \alpha, \quad (21)$$

This is obviously a different from of usual α-cut, because each α-level still retains uncertainty information interior of the interval that was generated by α.

5 | Numerical Example

In this part, in order to demonstrate the efficiency of the proposed model a numerical example with 3 (DCs), 2 products, 3 periods and 5 customer zones is generated accordance with the guidelines in the literature. The model is coded in LINGO 8.0 Software on a PC including two Intel® CoreTM2 and 2 GB RAM. Experiment results are indicated in *Table 1* and *Table 2*.

Table 1. Parameters.

B	**M**		**HC_w**		**U_p**	**PC_p**	**V_p**
Time units	Large number	(DCs)	Holding capacity	Product	Processing time	Processing cost	Volume
B=9000	M=1E+09	W_1	(3000,2800,3200)	P_1	5	(30,28,32)	1
		W_2	(3000,2800,3200)	P_2	10	(50,49,51)	2
		W_3	(3000,2800,3200)				

Table 2. Experiment results.

			HC_{pw}	**TC_{pw}**	**SC_{pt}**
Product	**Period**	**DC**	**Holding Cost**	**Transportation Cost from Plant to DCs**	**Set Up Cost**
P_1	T_1	W_1	(1, 0.5, 1.5)	(1,0.8,1.2)	(1000,800,1200)
P_1	T_2	W_2	(1,0.8, 1.2)	(2,1.8,1.2)	(1000,800,1200)
P_1	T_3	W_3	(1,0.7,1.7)	(3,2.9, 3.1)	(1000,800,1200)
P_2	T_1	W_1	(2,1.8,2.2)	(2,1.5, 2.5)	(1500,1400,1600)
P_2	T_2	W_2	(2,1.7,2.3)	(3,2.9, 3.1)	(1500,1400,1600)
P_2	T_3	W_3	(2,1.6,2.4)	(4,3.9, 4.1)	(1000,800,1200)

Table 3. Experiment results.

Product	**DC**	**Customer Zone**	**TC_{pwit} Transportation Cost from DCs to CZ**	**Customer Zone**	**Period**	**D_{pit} Demand**
P_1	W_1	I1	(5,4,6)	I1	T_1	108
P_1	W_1	I2	(7,6 ,7.5)	I1	T_2	177
P_1	W_1	I3	(10,9.5,11)	I1	T_3	174
P_1	W_1	I4	(11,10.25,11.2)	I2	T_1	121
P_1	W_1	I5	(15,14.2,15.8)	I2	T_2	74
P_1	W_2	I1	(5,4.5,5.1)	I2	T_3	182
P_1	W_2	I2	(7,6.8,7.15)	I3	T_1	83
P_1	W_2	I3	(10,9.4,10.1)	I3	T_2	116
P_1	W_2	I4	(11,10.9,11.3)	I3	T_3	198

Table 3. Continued.

Product	DC	Customer Zone	TC_{pwit} Transportation Cost from DCs to CZ	Customer Zone	Period	D_{pit} Demand
P_1	W_2	I5	(15,14.7,15.1)	I4	T_1	31
P_1	W_3	I1	(7,6.8,7.2)	I4	T_2	25
P_1	W_3	I2	(9,8.7,9.6)	I4	T_3	10
P_1	W_3	I3	(12,11.5,12.6)	I5	T_1	162
P_1	W_3	I4	(13,12.4,13.1)	I5	T_2	137
P_1	W_3	I5	(17,16.8,17.1)	I5	T_3	170
P_2	W_1	I1	(7,6.8,7.2)	I1	T_1	11
P_2	W_1	I2	(9,8.5,9.2)	I1	T_2	14
P_2	W_1	I3	(12,11.8,12.1)	I1	T_3	76
P_2	W_1	I4	(13,12.8,13.1)	I2	T_1	166
P_2	W_1	I5	(17,16.8,17.8)	I2	T_2	14
P_2	W_2	I1	(7,6.8,7.2)	I2	T_3	44
P_2	W_2	I2	(9,8.9,9.1)	I3	T_1	183
P_2	W_2	I3	(12,11.8,12.2)	I3	T_2	113
P_2	W_2	I4	(13,12.8,13.1)	I3	T_3	15
P_2	W_2	I5	(17,16.8,17.2)	I4	T_1	101
P_2	W_3	I1	(10,9.8,10.5)	I4	T_2	83
P_2	W_3	I2	(12,11.2,12.3)	I4	T_3	62
P_2	W_3	I3	(15,14.2,15.1)	I5	T_1	8
P_2	W_3	I4	(16,15.1,16.2)	I5	T_2	90
P_2	W_3	I5	(20,19.8,20.1)	I5	T_3	40

6 | Conclusion

SC design problems have recently raised a lot of interest since the opportunity of an integrated management of the SC can reduce the propagation of undesirable events through the network and can affect decisively the profitability of the members. Conventional SCM models deals with crisp or accurate data, however in real world problems accurate data may not be available. In many situations the data available are imprecise like fuzzy numbers. In the existing models uncertainties of the fuzzy data are effectively ignored through the computation. In this paper costs are considered as triangular fuzzy numbers and this paper proposes an alternative approach to retain fuzziness of the model by maximizing the membership functions of fuzzy data. The value of the objective function is equivalent to α-cut of interval programming problem with maximum possible value of the objective function.

For further studies, it is suggested to explore: 1) reducing the size of the converted (crisp equivalent) problem, and 2) possible linearization of the nonlinear model.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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References

[1] Melo, M. T., Nickel, S., & Saldanha-Da-Gama, F. (2009). Facility location and supply chain management--A review. *European journal of operational research*, *196*(2), 401–412. https://doi.org/10.1016/j.ejor.2008.05.007

[2] Gebennini, E., Gamberini, R., & Manzini, R. (2009). An integrated production–distribution model for the dynamic location and allocation problem with safety stock optimization. *International journal of production economics*, *122*(1), 286–304 https://doi.org/10.1016/j.ijpe.2009.06.027

[3] Aliev, R. A., Fazlollahi, B., Guirimov, B. G., & Aliev, R. R. (2007). Fuzzy-genetic approach to aggregate production–distribution planning in supply chain management. *Information sciences*, *177*(20), 4241–4255. https://doi.org/10.1016/j.ins.2007.04.012

[4] Fazel Zarandi, M. H., Fazel Zarani, M. M., & Saghiri, S. (2007). Five crisp and fuzzy models for supply chain of an automotive manufacturing system. *International journal of management science and engineering management*, *2*(3), 178–196. https://doi.org/10.1080/17509653.2007.10671020

[5] Bhatnagar, R., Chandra, P., & Goyal, S. K. (1993). Models for multi-plant coordination. *European journal of operational research*, *67*(2), 141–160. https://doi.org/10.1016/0377-2217(93)90058-

[6] Thomas, D. J., & Griffin, P. M. (1996). Coordinated supply chain management. *European journal of operational research*, *94*(1), 1–15. https://doi.org/10.1016/0377-2217(96)00098-7

[7] Vidal, C. J., & Goetschalckx, M. (1997). Strategic production–distribution models: a critical review with emphasis on global supply chain models. *European journal of operational research*, *98*(1), 1–18. https://doi.org/10.1016/S0377-2217(97)80080-X

[8] Beamon, B. M. (1998). Supply chain design and analysis: models and methods. *International journal of production economics*, *55*(3), 281–294. https://doi.org/10.1016/S0925-5273(98)00079-6

[9] Erengüç, Ș. S., Simpson, N. C., & Vakharia, A. J. (1999). Integrated production/distribution planning in supply chains: An invited review. *European journal of operational research*, *115*(2), 219–236. https://doi.org/10.1016/S0377-2217(98)90299-5

[10] Sarmiento, A. N. A. M., & Nagi, R. (1999). A review of integrated analysis of production-distribution systems. *IIE transactions*, *31*(11), 1061–1074. https://doi.org/10.1080/07408179908969907

[11] Lee, Y. H., & Kim, S. H. (2000, December). Optimal production-distribution planning in supply chain management using a hybrid simulation-analytic approach. *2000 winter simulation conference proceedings (Cat. No. 00CH37165)* (Vol. 2, pp. 1252-1259). IEEE. https://doi.org/10.1109/WSC.2000.899093

[12] Chandra, P., & Fisher, M. L. (1994). Coordination of production and distribution planning. *European journal of operational research*, *72*(3), 503–517. https://doi.org/10.1016/0377-2217(94)90419-7

[13] Jayaraman, V., & Pirkul, H. (2001). Planning and coordination of production and distribution facilities for multiple commodities. *European journal of operational research*, *133*(2), 394–408. https://doi.org/10.1016/S0377-2217(00)00033-3

[14] Lee, Y. H., & Kim, S. H. (2002). Production–distribution planning in supply chain considering capacity constraints. *Computers & industrial engineering*, *43*(1), 169–190. https://doi.org/10.1016/S0360-8352(02)00063-3

[15] Petrovic, D., Roy, R., & Petrovic, R. (1998). Modelling and simulation of a supply chain in an uncertain environment. *European journal of operational research*, *109*(2), 299–309. https://doi.org/10.1016/S0377-2217(98)00058-7

[16] Giannoccaro, I., Pontrandolfo, P., & Scozzi, B. (2003). A fuzzy echelon approach for inventory management in supply chains. *European journal of operational research*, *149*(1), 185–196. https://doi.org/10.1016/S0377-2217(02)00441-1

[17] Xie, Y., Petrovic, D., & Burnham, K. (2006). A heuristic procedure for the two-level control of serial supply chains under fuzzy customer demand. *International journal of production economics*, *102*(1), 37–50. https://doi.org/10.1016/j.ijpe.2005.01.016

[18] Chen, C.-T., Lin, C.-T., & Huang, S.-F. (2006). A fuzzy approach for supplier evaluation and selection in supply chain management. *International journal of production economics*, *102*(2), 289–301. DOI:https://doi.org/10.1016/j.ijpe.2005.03.009

[19] Chang, S.-L., Wang, R.-C., & Wang, S.-Y. (2006). Applying fuzzy linguistic quantifier to select supply chain partners at different phases of product life cycle. *International journal of production economics*, *100*(2), 348–359. https://doi.org/10.1016/j.ijpe.2005.01.002

[20] Xu, J., He, Y., & Gen, M. (2009). A class of random fuzzy programming and its application to supply chain design. *Computers & industrial engineering*, *56*(3), 937–950. https://doi.org/10.1016/j.cie.2008.09.045

[21] Selim, H., Araz, C., & Ozkarahan, I. (2008). Collaborative production–distribution planning in supply chain: A fuzzy goal programming approach. *Transportation research part e: logistics and transportation review*, *44*(3), 396–419. https://doi.org/10.1016/j.tre.2006.11.001

[22] Liang, T.-F., & Cheng, H.-W. (2009). Application of fuzzy sets to manufacturing/distribution planning decisions with multi-product and multi-time period in supply chains. *Expert systems with applications*, *36*(2, Part 2), 3367–3377. https://doi.org/10.1016/j.eswa.2008.01.002

[23] Bilgen, B. (2010). Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem. *Expert systems with applications*, *37*(6), 4488–4495. https://doi.org/10.1016/j.eswa.2009.12.062

[24] Mula, J., Peidro, D., & Poler, R. (2010). The effectiveness of a fuzzy mathematical programming approach for supply chain production planning with fuzzy demand. *International journal of production economics*, *128*(1), 136–143. https://doi.org/10.1016/j.ijpe.2010.06.007